

Distribution Amplitudes for the ρ meson

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Collaboration with J. R. Forshaw (University of Manchester)

JHEP 1011:037,2010 & Work in progress



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Diffractive ρ meson production at HERA



Recent and more precise data from HERA

- $\sigma = \sigma_L + \sigma_T$
- $d\sigma/dt$
- σ_L/σ_T
- $Q^2 \in [0, 36] \text{ GeV}^2$: includes photoproduction region

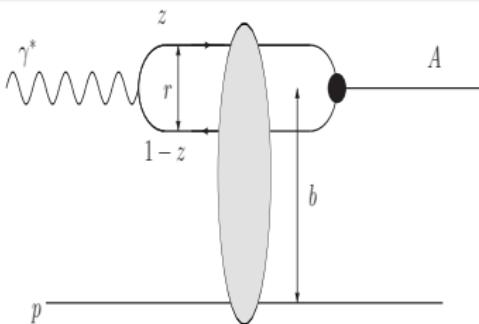
ZEUS Collaboration, PMC, Phys. A1 (2007) 6

H1 Collaboration, JHEP 12 (2010) 052

This work

Use data to extract information on the light-cone wavefunctions
and Distribution Amplitudes for the ρ

Colour dipole model



- $A = \rho$
- r : transverse dipole size
- z : fraction of photon's light-cone momentum carried by quark

At high energy ($s \gg t, Q^2, M_\rho^2$), amplitude factorises

$$\Im m \mathcal{A}_\lambda(s, t; Q^2) = \sum_{h, \bar{h}} \int d^2 \mathbf{r} dz \psi_{h, \bar{h}}^{\gamma^*, \lambda} \psi_{h, \bar{h}}^{\rho, \lambda*} e^{-iz\mathbf{r} \cdot \Delta} \mathcal{N}(x, \mathbf{r}, \Delta)$$

Universal dipole cross-section

$$\hat{\sigma}(x, \mathbf{r}) = \mathcal{N}(x, \mathbf{r}, \mathbf{0})/s$$

$\hat{\sigma}$ is well-constrained by very precise F_2 HERA data

Dipole models

Color Glass Condensate (CGC)-inspired

C. Marquet et al. Phys. Rev. D76 (2007) 034011

H. Kowalski and G. Watt, Phys. Rev. D78 (2008) 014016 ; G. Soyez, Phys. Lett. B655 (2007) 32

$$\begin{aligned}\mathcal{N}(rQ_s, x, 0) &= \mathcal{N}_0 \left(\frac{rQ_s}{2} \right)^{2\left[\gamma_s + \frac{\ln(2/rQ_s)}{\kappa \lambda \ln(1/x)}\right]} \quad \text{for} \quad rQ_s \leq 2 \\ &= \{1 - \exp[-a \ln^2(brQ_s)]\} \quad \text{for} \quad rQ_s > 2\end{aligned}$$

Saturation scale $Q_s = (x_0/x)^{\lambda/2}$

- CGC[0.63] : anomalous dimension $\gamma_s = 0.63$ (fixed)
- CGC[0.74] : anomalous dimension $\gamma_s = 0.74$ (fitted)

Non forward extension of CGC[0.74] : t-CGC

$$Q_s \rightarrow Q_s(t) = (x_0/x)^{\lambda/2} \times (1 + c\sqrt{|t|})$$



Dipole models

Regge-inspired (FSSat)

J. R. Forshaw and G. Shaw, JHEP 0412 (2004) 052

$r < r_0$: Hard Pomeron

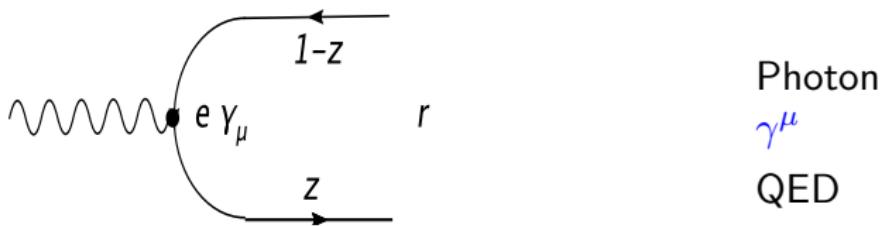
$$\hat{\sigma}^{\text{hard}}(x, r) = A_H r^2 x^{-\lambda_H}$$

$r > r_1$: Soft Pomeron

$$\hat{\sigma}^{\text{soft}}(x, r) = A_S x^{-\lambda_S}$$

- r_0 varies with $x \rightarrow$ saturation radius
- Linear interpolation for intermediate $r_0 < r < r_1$

Light cone wavefunctions



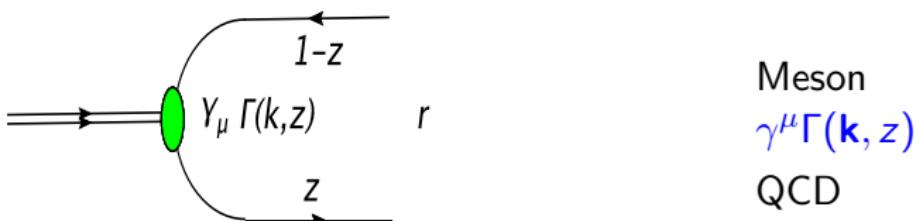
Spinor \times Scalar

$$\Psi_{h,\bar{h}}^{\gamma\{\lambda\}}(\mathbf{k}, z; Q^2) \propto S_{h,\bar{h}}^{\gamma,\lambda}(\mathbf{k}, z) \times \phi_\gamma(\mathbf{k}, z; Q^2)$$

$$S_{h,\bar{h}}^{\gamma,\lambda}(\mathbf{k}, z) = \frac{\bar{u}_h(\mathbf{k})}{\sqrt{z}} \gamma^\mu \cdot \varepsilon_\mu^\lambda \frac{v_{\bar{h}}(-\mathbf{k})}{\sqrt{1-z}}$$

- ▷ Sensitive to phenomenological quark mass m_f as $Q^2 \rightarrow 0$

Light cone wavefunctions



Spinor \times Scalar

$$\Psi_{h,\bar{h}}^{\rho\{\lambda\}}(\mathbf{k}, z) \propto S_{h,\bar{h}}^{\rho,\lambda}(\mathbf{k}, z) \times \phi_\rho(\mathbf{k}, z)$$

$$S_{h,\bar{h}}^{\rho,\lambda}(\mathbf{k}, z) = \frac{\bar{u}_h(\mathbf{k})}{\sqrt{z}} \gamma^\mu \cdot e_\mu^\lambda \frac{v_{\bar{h}}(-\mathbf{k})}{\sqrt{1-z}}$$

Model for scalar part

Constraints on meson wavefunction

Normalisation

$$1 = \sum_{h,\bar{h}} \int d^2\mathbf{r} dz |\Psi_{h,\bar{h}}^{\rho,\lambda}(r,z)|^2 \equiv \int d^2\mathbf{r} dz |\Psi^{\rho,\lambda}(r,z)|^2$$

Leptonic decay width

$$f_\rho M_\rho = \frac{N_c}{\pi} e_f \int_0^1 \frac{dz}{z(1-z)} [z(1-z)M_\rho^2 + m_f^2 - \nabla_r^2] \phi_L(r,z) \Big|_{r=0}$$

Boosted Gaussian

Simplified version of wavefunction by Nikolaev, Nemchik, Predazzi and Zakharov

$$\phi_{\lambda}^{\text{BG}}(r, z) = \mathcal{N}_{\lambda}[z\bar{z}] \exp\left(-\frac{m_f^2 R_{\lambda}^2}{8[z\bar{z}]}\right) \exp\left(-\frac{2[z\bar{z}]r^2}{R_{\lambda}^2}\right)$$

- R_{λ} and \mathcal{N}_{λ} fixed using normalisation and decay width constraints
- OK for earlier HERA data
- Cannot fit new data with any of the CGC or FSSat models

Scalar wavefunction

Boosted Gaussian

$$\phi_\lambda^{\text{BG}}(r, z) = \mathcal{N}_\lambda [z\bar{z}]^{b_\lambda} \exp\left(-\frac{m_f^2 R_\lambda^2}{8[z\bar{z}]^{b_\lambda}}\right) \exp\left(-\frac{2[z\bar{z}]^{b_\lambda} r^2}{R_\lambda^2}\right)$$

- Allow b_λ to vary freely
- This enhances end-point contribution

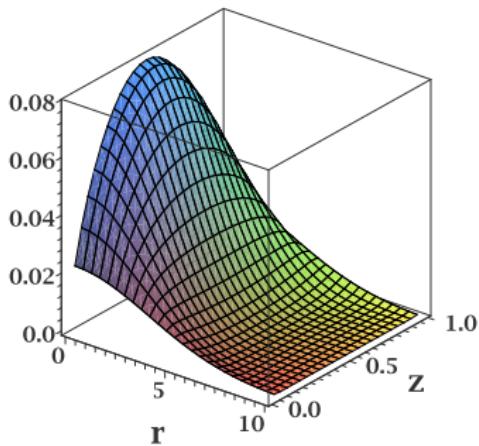
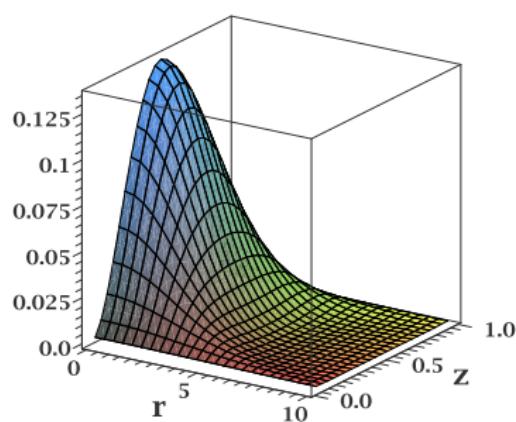
Additional enhancement

$$\phi_\lambda(r, z) = \phi_\lambda^{\text{BG}}(r, z) \times [1 + c_\lambda \xi^2 + d_\lambda \xi^4]$$

$$\xi \equiv 2z - 1$$

Extracted wavefunctions with FSSat

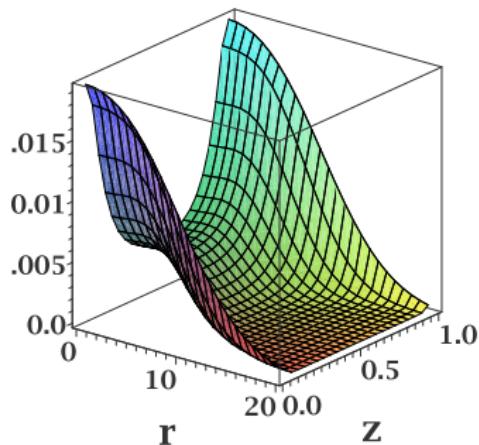
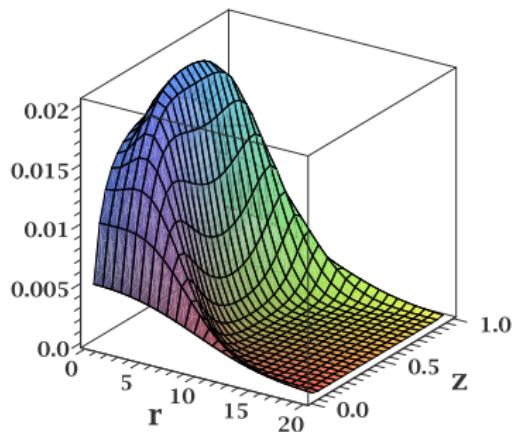
J. R. Forshaw and R. Sandapen, JHEP 1011 :037, 2010



- Longitudinal polarisation
- Mild end-point enhancement

Extracted wavefunctions with FSSat

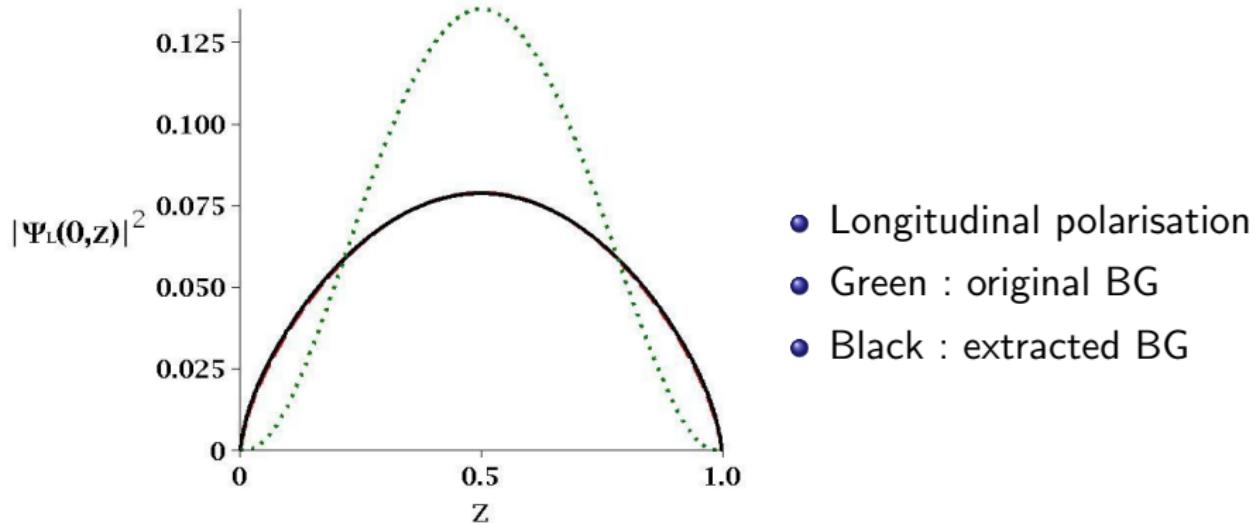
J. R. Forshaw and R. Sandapen, JHEP 1011 :037, 2010



- Transverse polarisation
- Significant end-point enhancement

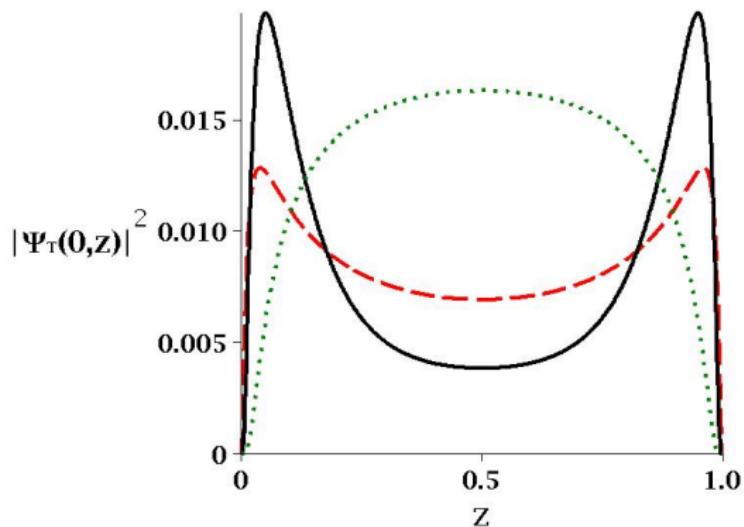
Extracted wavefunctions with FSSat

JHEP 1011 : 037,2010



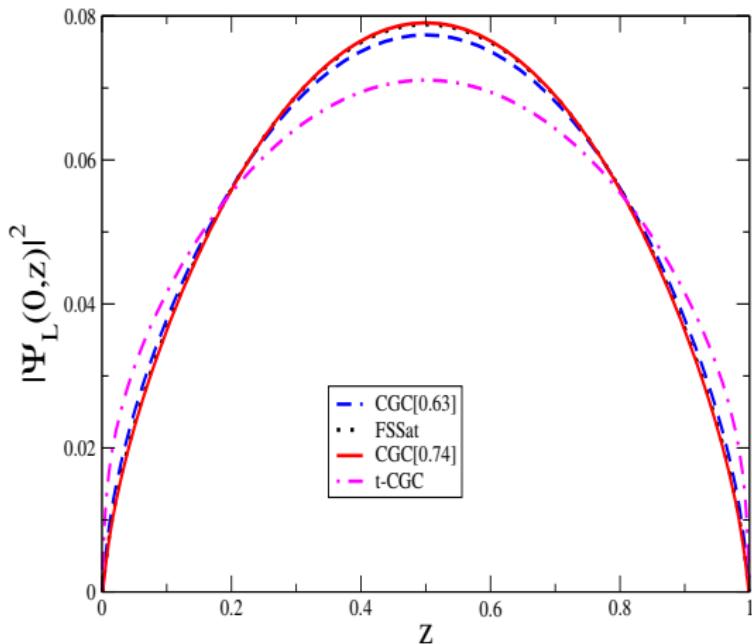
Extracted wavefunctions with FSSat

JHEP 1011 : 037,2010



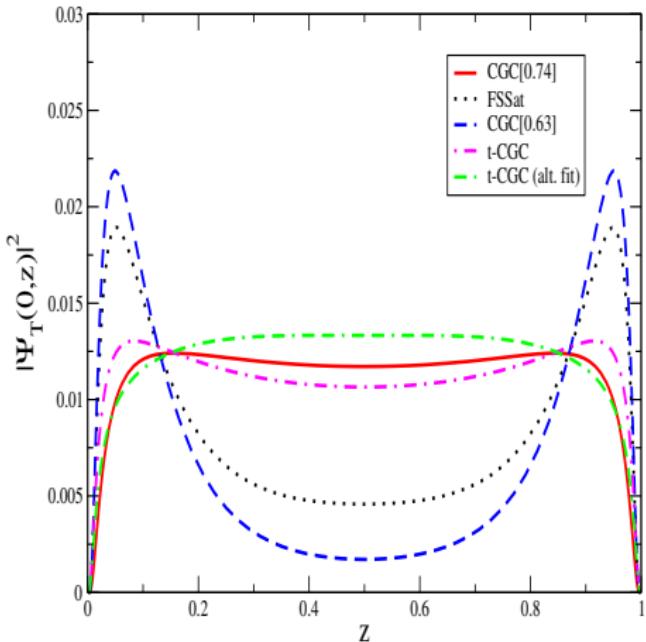
- Transverse polarisation
- Green : original BG
- Red : extracted BG
 $\chi^2/\text{d.o.f} = 82/70$
- Black : extracted BG
 $\chi^2/\text{d.o.f} = 68/72$
- Additional end-point enhancement required

Extracted wavefunctions with CGC models



- Longitudinal polarisation
- Not much model dependent
- t-CGC : slightly broader

Extracted wavefunctions with CGC models



- Transverse polarisation
- No additional enhancement required with CGC[0.74] and t-CGC
- All wavefunctions broader than original BG

Distribution Amplitudes

Meson-to-vacuum matrix elements on the light-cone

Ball, Braun et Lenz, JHEP 08 (2007) 090

$$\begin{aligned} \langle 0 | \bar{q}(0)[0, x] \gamma_\mu q(x) | \rho(P, \lambda) \rangle \propto & \left\{ \frac{e^{(\lambda)} \cdot x}{P \cdot x} P_\mu \int_0^1 du e^{-iuP \cdot x} \phi_{\parallel}(u, \mu) \right. \\ & \left. + \left(e_\mu^{(\lambda)} - P_\mu \frac{e^{(\lambda)} \cdot x}{P \cdot x} \right) \int_0^1 du e^{-iuP \cdot x} g_{\perp}(u, \mu) \right\} \end{aligned}$$

Twist classification

$$\text{Twist-2 : } \phi_{\parallel}(z, \mu) \propto \int dx^- e^{izP^+x^-} \langle 0 | \bar{q}(0) e^{L^*} \cdot \gamma q(x^-) | \rho(P, L) \rangle$$

$$\text{Twist-3 : } g_{\perp}(z, \mu) \propto \int dx^- e^{izP^+x^-} \langle 0 | \bar{q}(0) e^{T^*} \cdot \gamma q(x^-) | \rho(P, T) \rangle$$

Distribution Amplitudes

Explicit forms

Twist-2 :

$$\phi_{\parallel}(z, \mu) = 6z(1-z) \left[1 + a_2^{\parallel}(\mu) \frac{3}{2} (5\xi^2 - 1) \right]$$

Twist-3 :

$$\begin{aligned} g_{\perp}(z, \mu) &= \frac{3}{4}(1 + \xi^2) + \left(\frac{3}{7}a_2^{\parallel}(\mu) + 5\zeta_3(\mu) \right) (3\xi^2 - 1) \\ &+ \left[\frac{9}{112}a_2^{\parallel}(\mu) + \frac{15}{64}\zeta_3(\mu) \left(3\omega_3^V(\mu) - \omega_3^A(\mu) \right) \right] \\ &\times (3 - 30\xi^2 + 35\xi^4) \end{aligned}$$

$$\xi \equiv 2z - 1$$

Distribution Amplitudes

QCD Sum Rules and evolution

- QCD Sum Rules to estimate parameters at $\mu = 1$ GeV
- pQCD evolution
- Parameters vanish as $\mu \rightarrow \infty$

Asymptotic DAs

Twist-2 :

$$\phi_{\parallel}(z, \infty) = 6z(1 - z)$$

Twist-3 :

$$g_{\perp}(z, \infty) = \frac{3}{4}(1 + \xi^2)$$

Connection with Distribution Amplitudes

Identify renormalization scale μ with cut-off on \mathbf{k}

$$\int dx^- e^{-ix^- zP^+} \langle 0 | \bar{q}(0) e^{\lambda*} \cdot \gamma q(x^-) | \rho(P, \lambda) \rangle = \\ \int^{|\mathbf{k}| < \mu} \frac{d^2 \mathbf{k}}{16\pi^3} \sum_{h, \bar{h}} |S_{h, \bar{h}}^{\rho, \lambda}(z, \mathbf{k})|^2 \phi_\lambda(z, \mathbf{k})$$

Explicitly

$$\phi_{||}(z, \mu) = \frac{M_\rho}{\pi f_\rho} \int^{|\mathbf{k}| < \mu} \frac{d^2 \mathbf{k}}{4\pi^2} \phi_L(z, \mathbf{k})$$

$$g_\perp(z, \mu) = \frac{1}{2\pi f_\rho M_\rho} \int^{|\mathbf{k}| < \mu} \frac{d^2 \mathbf{k}}{4\pi^2} (m_f^2 + (z^2 + (1-z)^2) \mathbf{k}^2) \frac{\phi_T(z, \mathbf{k})}{z(1-z)}$$

Connection with Distribution Amplitudes

In coordinate space

Twist-2

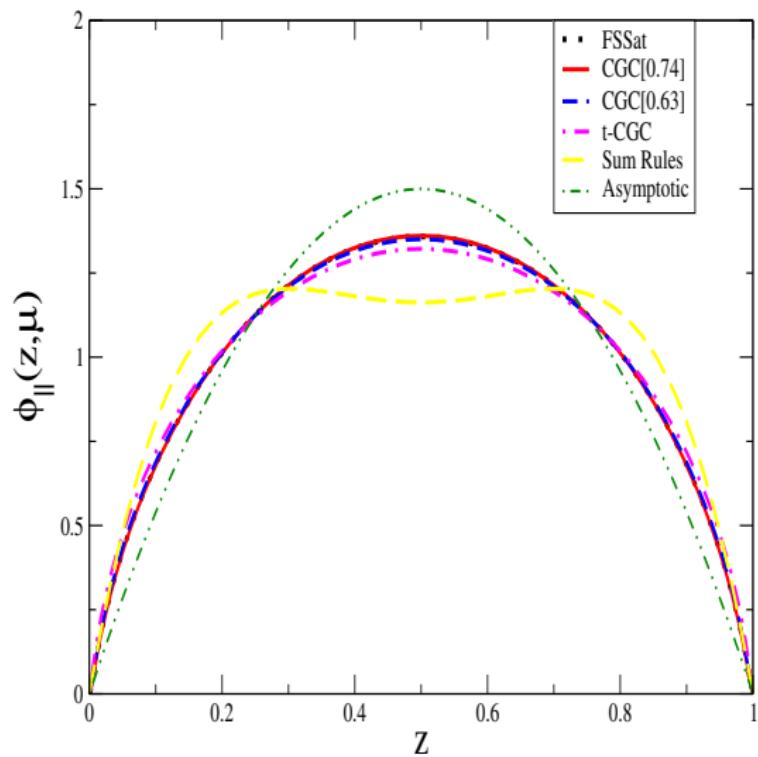
$$\phi_{\parallel}(z, \mu) = \frac{1}{2\pi f_\rho} \int dr \mu J_1(\mu r) M_\rho \phi_L(r, z)$$

Twist-3

$$g_{\perp}(z, \mu) = -\frac{1}{2\pi f_\rho M_\rho} \int dr \mu J_1(\mu r) [(z^2 + \bar{z}^2) \nabla_r^2 - m_f^2] \frac{\phi_T(r, z)}{(z\bar{z})^2}$$

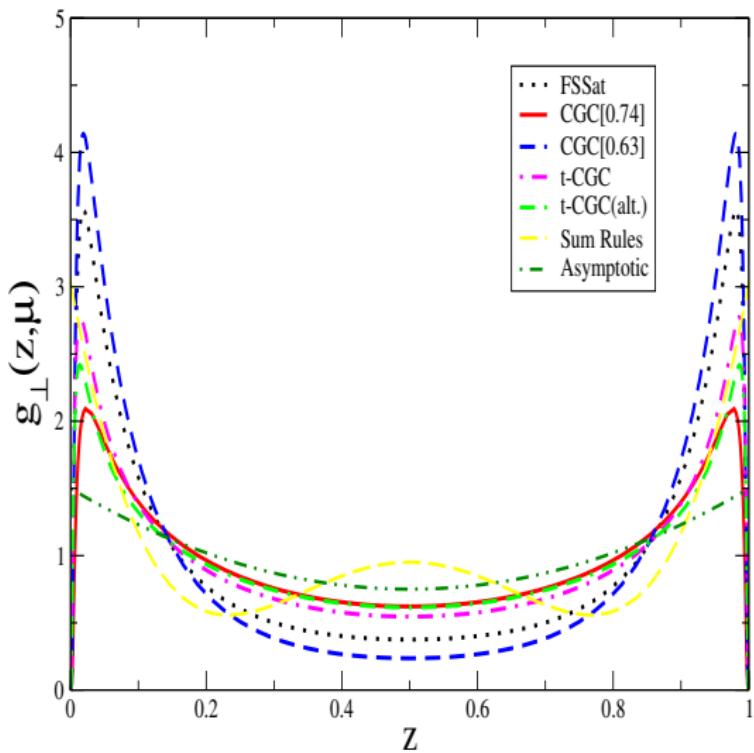
$$\bar{z} \equiv 1 - z$$

Extracted Distribution Amplitudes



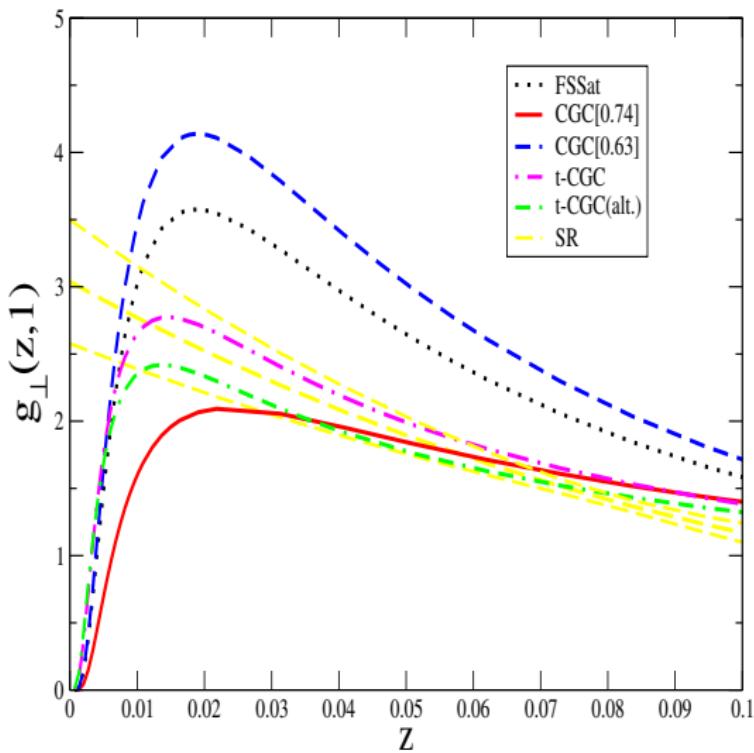
- Twist-2 DAs at $\mu = 1$ GeV
- Agreement with Sum Rules
- Non-monotonic due to truncation in Gegenbauer expansion
- Dark green : asymptotic

Extracted Distribution Amplitudes



- Twist-3 DAs at $\mu = 1$ GeV
- Model-dependent
- Dark green : asymptotic

Extracted Distribution Amplitudes



- Sum Rule DA with error band
- Prefers DAs extracted with t-CGC

Moment of the twist-2 DA

Lowest moment

$$\langle \xi^2 \rangle_\mu = \int_0^1 dz \xi^2 \varphi(z, \mu)$$

Approach	Scale μ	$\langle \xi^2 \rangle_\mu$
Old BG prediction	~ 1 GeV	0.181
CGC[0.74]	~ 1 GeV	0.227
CGC[0.63]	~ 1 GeV	0.229
t-CGC	~ 1 GeV	0.236
FSSat	~ 1 GeV	0.227
Sum Rules	1 GeV	0.254
Lattice	2 GeV	0.24(4)

Conclusions

- Current data require qualitatively different light-cone wavefunctions for transverse and longitudinal polarisation
- Extracted twist-2 DA not much model dependent
- Agrees with QCD Sum Rules and lattice predictions
- Twist-3 DAs extracted in the t-CGC model are preferred by Sum Rules predictions
- All extracted DAs are broader than the asymptotic forms